WG14: University mathematics education

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Research on university level mathematics education is a relatively young field, which embraces an increasingly wider range of *theoretical approaches* (e.g. cognitive/developmental, socio-cultural, anthropological and discursive) and *methods/methodologies* **(**e.g. quantitative, qualitative and narrative). Variation also characterises research in this area with regard to at least two further issues: the *role of the participants*, students and university teachers, in the research – from 'just' subjects of the research to fully-fledged co-researchers – and the *degree of intervention* involved in the research – from external, non-interventionist research, to developmental/action research in which researchers identify problems and devise, implement and evaluate reforms of practice (Artigue et al, 2007).

2011 marks the 20th anniversary of the publication of *Advanced Mathematical Thinking* (Tall, 1991), a volume that is often heralded as a first signal of the emergence of this new area of research. A few years later a second signal was given by the 1998 ICMI study that resulted in *The teaching and learning of mathematics at university level* (Holton, 2001). In the meantime Advanced Mathematical Thinking (AMT) groups ran both in previous CERME and PME conferences; sessions exclusively on university mathematics education are part of the EMF conferences since 2006 ; the RUME, UMT and Delta conferences emerged in the USA, the UK and South Africa respectively; the International Conferences on the Teaching of Mathematics at University Level were launched in 1998; etc.

WG14 (*University Mathematics Education*, hereafter UME) emerged out of the above developments and out of the realisation that this is a distinct area of mathematics education research. Its distinctiveness can be attributed to several characteristics. Firstly, the classic distinction between 'teacher' and 'researcher' does not always apply in UME as researchers in mathematics education in this area are often university-level teachers of mathematics themselves. In particular there is a growing group of mathematicians specializing in research on mathematics education at university level, where advanced mathematical knowledge and experience is really an asset (if not a necessity). Secondly, mathematics education theories and research methods find new uses, and adaptations, at the university level. These adaptations are often quite radical as the post-compulsory educational context is different in many ways – the voluntary presence of students, the important role of mathematics as a service subject, the predominance of lecturing to large numbers of students, the absence of national programmes for university education, to mention but a few. In this sense UME is a distinct area of mathematics education research, not merely a

mirror of mathematics education research at a more advanced educational level. Finally, in recent years, research in this area has been growing in different parts of the world. WG14 is one opportunity to collate evidence of this growing research activity from Europe and beyond.

The WG14 Call for Papers invited contributions from as wide a range of research topics as possible. These included: concept formation, mathematical reasoning and proof in university mathematics; teaching at university level (including the perspectives of university mathematicians); ICT in the teaching and learning of university mathematics (including curriculum design); socio-cultural, anthropological and discursive approaches in university mathematics education; affective and social aspects of the learning and teaching of university mathematics; the transition from secondary to tertiary mathematics; novel approaches to teaching mainstream (e.g. calculus and linear algebra) as well as more advanced topics, both to students of mathematics and other areas (e.g. engineering, sciences, finance); theoretical approaches to the study of teaching and learning mathematics at university (including a focus on specific approaches and on contrasting or combining approaches).

This report draws on the presentations, reactions-to and discussions of the 21 accepted papers that met these terms. The number and quality of these papers marks the recent surge in the quality of research outputs and a move away from the earlier days of perhaps more naïve, less rigorous research in this area – brought about partly by the university sector's increasing urge to adapt teaching to changing student cohorts and by a growing, and wider, tendency towards an in-depth probing into traditional teaching practices in higher education.

Across the WG14 discussions certain themes and questions emerged as crucial. These included: exploring whether UME needs to generate new theories or adapt already existing ones; attending to issues of both theory and practice; acknowledging that research on teaching and learning in higher education develops also outside mathematics education, and benefiting from these developments; working towards the generation of new theories while valuing already accumulated knowledge in the field; etc. Colleagues observed that, beyond staple references to classic constructs from the *AMT* era (such as concept image – concept definition; APOS theory, process – object duality etc.), several works presented in WG14 employ (often in tandem with the above) approaches such as the Anthropological Theory of Didactics (ATD: Chevallard, 1985), and discursive approaches (e.g. Sfard, 2008).

Generally speaking papers seem to be classified into those with a focus on the teaching and learning of particular mathematical topics (calculus continues to attract more attention than other topics) or on wider, cross-topical issues such as the transition to university mathematics, use of IT, language, motivation, teacher knowledge and development, curricular, pedagogical and institutional issues, etc.. Furthermore an area of growth is of studies that examine the different role of mathematics in courses towards a mathematics degree, courses for pre-service teachers, as a 'service' subject (physics, biology, economics etc.). While a substantial

number of papers remains in the increasingly well-trodden area of students' perceptions of specific mathematical concepts (again calculus prevails in these), a focus on university teachers and teaching is also emerging, if often a little timidly, and diplomatically, resulting in descriptive, openly non-judgemental studies. In conjunction with those studies a genre of collaborative studies, with mathematicians engaged as co-researchers, also seems to be on the rise. In the nutshell descriptions of the WG14 papers that follow the order of presentation is loosely structured around some of the themes mentioned above.

Xhonneux & Henry is one of the papers that employs the ATD framework to distinguish between *mathematical* and *didactic* praxeologies in the context of teaching and learning of Lagrange's Theorem in calculus courses to mathematics and economics students. *Gyöngyösi, Solovej & Winslow* is another: in it a part of a transitional course in Analysis was taught with a combination of Maple and paper-based techniques and resulted in mixed reception and performance by students. A third is *Barquero, Bosch & Gascón*: from its analyses 'applicationism' emerges as the prevailing epistemology of mathematics in science departments that potentially hinders the teaching of mathematical modelling to science students.

Another set of approaches that was employed by a number of WG14 papers were discursive. *Jaworski & Matthews* employed such approaches to trace university mathematicians' pedagogical discourse and suggest links of this discourse to their ontological and epistemological perspectives. *Biza & Giraldo* described how computational inscriptions – in this case differentiability – have potentialities and limitations that can be helpful in students' exploration of newly introduced mathematical concepts. Three papers made use of Sfard's commognitive framework. *Viirman* employed this framework to trace the variation in the pedagogical discourses of mathematics lecturers in the course of their introducing the concept of function. *Stadler* described students' experience of the transition from school to university mathematics as an often perplexing re-visiting of content and ways of working that seem simultaneously familiar and novel (for example in the case of solving equations). *Nardi* outlined interviewed mathematicians' perspectives on their newly arriving students' verbalisation skills; and, observed that discourse on verbalisation in mathematics tends to be risk-averse and not as explicit in teaching as necessary.

Several papers focused on the transition from school to university mathematics (including *Gyöngyösi, Solovej & Winslow* and *Stadler* mentioned above). *Biehler, Fischer, Hochmuth & Wassong* proposed that blending traditional course attendance with systematic e-learning study can facilitate the bridging of school and university mathematics. *Faulkner, Hannigan & Gill* noted the shifting profile of students who take service mathematics courses (in the context of an Irish institution): many more are diagnosed as at risk, fewer have an advanced mathematics secondary qualification and the percentage of non-standard (e.g. mature) students has grown. *Zimmermann, Bescherer & Spannagel* described MaSE-T, a mathematics self-efficacy test designed to measure the impact that self-efficacy perceptions have on choice of studies.

Vandebrouck noted that in the transition from school to university, mathematics students need to reconceptualise the concept of function in terms of its multiple representations and its process-object duality. Finally, *De Vleeschouwer & Gueudet* observed that students can learn to appreciate the duality in linear forms (described here in micro-macro terms) if given an appropriate set of tasks that require them to engage with these concepts at both levels.

Many of the papers mentioned above had a clear focus on a specific mathematical concept or issue. In addition to these, *Iannone & Inglis* discussed a range of weaknesses in Year 1 mathematics students' production of deductive arguments (rather than in the oft-reported perception that a deductive argument was expected of them). *Juter* reported the highly individual and not easily classifiable character of pre-service secondary mathematics teachers' concept images of elementary Calculus concepts. And *Souto-Rubio & Gómez-Chacón* mapped out students' difficulty with developing visualisation skills in the context of the Riemann integral.

Some papers focused on particular curricular and pedagogical aspects of university mathematics. *Agathokleous* argued how teaching Abstract Algebra to pre-service primary teachers can facilitate students' appreciation for the connectedness across mathematical domains. *Jukic & Dahl*, through data collected in the Croatian and Danish context, aimed to illustrate that students taught in different styles are likely to perform differently. *Bergsten* presented evidence that students tend to find lectures useful and attractive, despite their bad press in some education quarters.

Finally, a few papers addressed theoretical issues directly. *Barton* outlined efforts to combine the three-fold activity of research, development and theory building into LATUM, a model for learning and teaching university mathematics that is proposed as a model for designing alternative university mathematics delivery. And, *Pettersson* proposed 'threshold concepts', a theoretical construct from the general education literature, as a means for gaining insight into student learning and engaging teachers in pedagogical discussion.

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